Computers and Mathematics with Applications 64 (2012) 1532–1544

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Further results for robust stability of homogeneous large-scale bilinear systems with time delays and uncertainties

Chien-Hua Lee, Cheng-Yi Chen *

Department of Electrical Engineering, Cheng Shiu University, Kaohsiung 83347, Taiwan

ARTICLE INFO

Keywords:
Homogeneous bilinear system
Time-delay
Uncertainty
Constrained input
Transient response

ABSTRACT

In this paper, the problem of stability analysis for homogeneous large-scale time-delay bilinear systems subjected to uncertainties is considered. Two classes of uncertainty are treated: nonlinear uncertainties and parametric uncertainties. By making use of the Lyapunov stability approach associated with solution bounds of the Lyapunov equation, two delay-independent criteria are presented to guarantee the robust stability of the overall systems. The stability condition for the mentioned system with nonlinear uncertainties is sharper than that of a previous work. The main feature of the schemes presented is that they do not involve any Lyapunov equation which may be unsolvable although the Lyapunov stability theorem is used.

1. Introduction

From the literature, it is well known that a bilinear structure can model nonlinear phenomena more accurately than a linear structure can [1–6]. Therefore, bilinear systems arise naturally as models for various dynamical processes, such as nuclear fission, chemical reactions, fluid flows, distillation columns, and wastewater treatment, especially in chemical engineering processes [4–6]. Furthermore, due to their theoretical and practical importance, a number of contributions have been devoted to the study of bilinear systems for several decades [7–25]. Basically, such research can be classified into two important topics: the stability analysis problem and stabilizing controller design. The former has been studied in [13,15,21,22] and the latter has been proposed in [4,7–10,12,13,16–20,23,25]. In addition, the stability analysis of large-scale bilinear systems has been studied in [13,14,19,24]. In practical considerations, time delays and uncertainties are the two important factors that can change the behavior of the characteristic equation and result in unsatisfactory performance or even unstable systems. Therefore, they should be integrated into system models. In the literature, studies of bilinear systems subjected to time delays and/or uncertainties have been of great interest. However, it seems that none of the existing works has discussed continuous large-scale bilinear systems with time delays and uncertainties except [11]. In [11], the robust stability of continuous large-scale bilinear systems subjected to uncertainties and time delays has been discussed. Several delay-independent criteria that ensure the robust stability of the overall systems were proposed. However, as mentioned in [11], those results obtained that involve matrix measures are somewhat conservative. Therefore, the objective of this paper is to improve the conservation of those criteria proposed in [11]. By using the Lyapunov equation approach associated with solution bonds of the Lyapunov equation, several robust stability conditions are established. It is shown that the condition for the system with nonlinear uncertainties mentioned is sharper than that of [11]. Furthermore, the criteria obtained for the interval system is also better than that proposed in [11] under some given assumptions. Finally, we give numerical examples to demonstrate the merits of the proposed results.

* Corresponding author. Tel.: +886 7 7310606; fax: +886 7 7337390.
E-mail address: albert@csu.edu.tw (C.-Y. Chen).

0898-1221/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2012.03.108

The following symbol conventions are used in this paper. Symbols $\boldsymbol{R}, A^T, \lambda_1(A), x^T(t), \|x(t)\|$, and $\|A\|$, respectively, mean real number field, transpose of matrix A, the maximal eigenvalue of a symmetric matrix A, transpose of vector $x(t)$, norm of vector $x(t)$ with $\|x(t)\| = \left(x^T(t)x(t)\right)^{1/2}$, and induced norm of matrix A with $\|A\| = \lambda_1(A^TA)^{1/2}$. Furthermore, $\mu(A)$ represents the matrix measure of A and is defined as $\mu(A) = \lambda_1((A + A^T)/2)$, $|A| = \left\{|a_{ij}|\right\}$ for matrix $A = \left\{a_{ij}\right\}$, and $A \geq 0$ means that the symmetric matrix A is positive semi-definite.

2. Systems with nonlinear uncertainties

Consider the following homogeneous large-scale bilinear time-delay system $\boldsymbol{S}$ subjected to nonlinear uncertainties which is described as an interconnection of N subsystems $S_1, S_2, \ldots, S_N$, which are represented by

$$S_i : \dot{x}_i(t) = A_ix_i(t) + \sum_{j=1}^{N} f_{ij}(x_j(t - d_{ij}), t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}x_j(t - d_{ij}) + \sum_{k=1}^{m_i} \operatorname{sat} u_{ik}(t)B_{ik}x_i(t) \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i(\cdot) \in \boldsymbol{R}^n$ and $u_{ik}(\cdot) \in \boldsymbol{R}^m$ are the state vector and the input, respectively; A_i, A_{ij}, and B_{ik} represent constant matrices with appropriate dimensions; $d_{ij} > 0$ for all i and j with $d_{ii} = 0$ denote the communication delays in the interconnections; and $f_{ij}(x_j(t - d_{ij}), t)$ represent nonlinear uncertainties possessing the following properties:

$$\|f_{ij}(x_j(t - d_{ij}), t)\| \leq \varepsilon_{ij}\|x_j(t - d_{ij})\|, \tag{2}$$

where $\varepsilon_{ij} > 0$ are constants. The inputs $\operatorname{sat} u_{ik}(t)$ are saturating functions, defined as follows.

$$\operatorname{sat} u_{ik}(t) = \begin{cases} u_{ik}(t), & \text{if } |u_{ik}(t)| \leq U_{ik} \\ U_{ik}\operatorname{sgn}(u_{ik}(t)), & \text{if } |u_{ik}(t)| > U_{ik} > 0, \end{cases} \tag{3}$$

where U_{ik} are positive constants. From (3), we have

$$|\operatorname{sat} u_{ik}(t)| \leq U_{ik}, \quad k = 1, 2, \ldots, m_i. \tag{4}$$

Then, by using the Lyapunov equation approach associated with linear algebraic techniques, we have the following result.

Theorem 1. *For $i = 1, 2, \ldots, N$, if there are positive constants q_i, α_{ji}, β_{ij}, and γ_{ik} such that the following condition is satisfied*

$$A_i^T + A_i + \left[\sum_{j=1}^{N}\left(\frac{q_j}{q_i\beta_{ji}}\varepsilon_{ji}^2 + \beta_{ij}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{N}\alpha_{ij} + \sum_{k=1}^{m_i}\gamma_{ik}\right]I + \sum_{\substack{j=1 \\ j\neq i}}^{N}\frac{q_j}{q_i\alpha_{ji}}A_{ji}^TA_{ji} + \sum_{k=1}^{m_i}\frac{1}{\gamma_{ik}}U_{ik}^2B_{ik}^TB_{ik} < 0, \tag{5}$$

then the large-scale uncertain bilinear time-delay system (1) with constraints (3) is robustly stable.

Proof. For convenience, we use symbols x_i, u_{ik}, and f_{ij} to represent $x_i(t)$, $u_{ik}(t)$, and $f_{ij}(x_j(t-d_{ij}), t)$ for all i and j, respectively, in the following and later descriptions. Condition (5) infers that $A_i^T + A_i < 0$, which means that matrix A_i is stable. Then, for a given positive definite matrix Q_i, the Lyapunov equation

$$A_i^TP_i + P_iA_i = -Q_i, \quad i = 1, 2, \ldots, N \tag{6}$$

has a unique positive definite solution P_i. We now choose the positive definite matrix Q_i as

$$Q_i = q_i\left\{\left[\sum_{j=1}^{N}\left(\frac{q_j}{q_i\beta_{ji}}\varepsilon_{ji}^2 + \beta_{ij}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{N}\alpha_{ij} + \sum_{k=1}^{m_i}\gamma_{ik}\right]I + \sum_{\substack{j=1 \\ j\neq i}}^{N}\frac{q_j}{q_i\alpha_{ji}}A_{ji}^TA_{ji} + \sum_{k=1}^{m_i}\frac{1}{\gamma_{ik}}U_{ik}^2B_{ik}^TB_{ik}\right\}, \tag{7}$$

where q_i, α_{ij}, β_{ij}, and γ_{ik} are positive constants. Then we rewrite (6) as

$$A_i^T(q_iI - P_i) + (q_iI - P_i)A_i = q_i(A_i^T + A_i) + Q_i, \quad i = 1, 2, \ldots, N. \tag{8}$$

Substituting (7) into (8) gives

$$\begin{aligned} A_i^T(q_iI - P_i) + (q_iI - P_i)A_i = q_i\Bigg\{&A_i^T + A_i + \left[\sum_{j=1}^{N}\left(\frac{q_j}{q_i\beta_{ji}}\varepsilon_{ji}^2 + \beta_{ij}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{N}\alpha_{ij} + \sum_{k=1}^{m_i}\gamma_{ik}\right]I \\ &+ \sum_{\substack{j=1 \\ j\neq i}}^{N}\frac{q_j}{q_i\alpha_{ji}}A_{ji}^TA_{ji} + \sum_{k=1}^{m_i}\frac{1}{\gamma_{ik}}U_{ik}^2B_{ik}^TB_{ik}\Bigg\}. \end{aligned} \tag{9}$$

It is seen that, if condition (5) is satisfied, then the right-hand side of (9) is a negative definite matrix. Therefore, Eq. (9) is a Lyapunov equation and has a positive definite solution; that is,

$$q_i I - P_i > 0, \quad i = 1, 2, \ldots, N. \tag{10}$$

This means that the solution P_i of (6) has the upper bound

$$P_i < q_i I, \quad i = 1, 2, \ldots, N. \tag{11}$$

We choose the Lyapunov function $V(x_i(t), t)$ for large-scale system (1) as

$$\begin{aligned} V(x_i(t), t) &= \sum_{i=1}^{N} V_i(x_i(t), t) \\ &= \sum_{i=1}^{N} \left\{ x_i^T P_i x_i + \sum_{\substack{j=1 \\ j\neq i}}^{N} \int_{t-d_{ij}}^{t} x_j^T(s) \left[\frac{1}{\alpha_{ij}} A_{ij}^T P_i A_{ij} + \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 I \right] x_j(s) ds \right\}, \end{aligned} \tag{12}$$

where P_i satisfies Lyapunov equation (6). Taking the derivative of $V(x_i(t), t)$ along trajectories of (1) results in

$$\begin{aligned} \dot{V}(x_i(t), t) &= \sum_{i=1}^{N} \left\{ \dot{x}_i^T P_i x_i + x_i^T P_i \dot{x}_i + \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T \left[\frac{1}{\alpha_{ij}} A_{ij}^T P_i A_{ij} + \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 I \right] x_j \right. \\ &\quad \left. - \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T(t-d_{ij}) \left[\frac{1}{\alpha_{ij}} A_{ij}^T P_i A_{ij} + \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 I \right] x_j(t-d_{ij}) \right\} \\ &= \sum_{i=1}^{N} \left\{ \left[A_i x_i + \sum_{j=1}^{N} f_{ij} + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij} x_j(t-d_{ij}) + \sum_{k=1}^{m_i} \operatorname{sat} u_{ik} B_{ik} x_i \right]^T P_i x_i \right. \\ &\quad + x_i^T P_i \left[A_i x_i + \sum_{j=1}^{N} f_{ij} + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij} x_j(t-d_{ij}) + \sum_{k=1}^{m_i} \operatorname{sat} u_{ik} B_{ik} x_i \right] \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T \left[\frac{1}{\alpha_{ij}} A_{ij}^T P_i A_{ij} + \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 I \right] x_j \\ &\quad \left. - \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T(t-d_{ij}) \left[\frac{1}{\alpha_{ij}} A_{ij}^T P_i A_{ij} + \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 I \right] x_j(t-d_{ij}) \right\} \\ &= \sum_{i=1}^{N} \left\{ x_i^T (A_i^T P_i + P_i A_i) x_i + \sum_{j=1}^{N} f_{ij}^T P_i x_i + x_i^T P_i \sum_{j=1}^{N} f_{ij} + \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T(t-d_{ij}) A_{ij}^T P_i x_i \right. \\ &\quad + x_i^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j(t-d_{ij}) A_{ij} + x_i^T \left[\sum_{k=1}^{m_i} \operatorname{sat} u_{ik} B_{ik}^T P_i + P_i \sum_{k=1}^{m_i} \operatorname{sat} u_{ik} B_{ik} \right] x_i \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T \left[\frac{1}{\alpha_{ij}} A_{ij}^T P_i A_{ij} + \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 I \right] x_j \\ &\quad \left. - \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T(t-d_{ij}) \left[\frac{1}{\alpha_{ij}} A_{ij}^T P_i A_{ij} + \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 I \right] x_j(t-d_{ij}) \right\}. \end{aligned} \tag{13}$$

We have

$$\sum_{j=1}^{N} f_{ij}^T P_i x_i + x_i^T P_i \sum_{j=1}^{N} f_{ij} \le x_i^T \sum_{j=1}^{N} \beta_{ij} P_i x_i + \sum_{j=1}^{N} \frac{1}{\beta_{ij}} f_{ij}^T P_i f_{ij} \tag{14}$$

$$\sum_{\substack{j=1\\ j\neq i}}^{N} x_j^T(t-d_{ij})A_{ij}^T P_i x_i + x_i^T P_i \sum_{\substack{j=1\\ j\neq i}}^{N} x_j(t-d_{ij})A_{ij} \le x_i^T \sum_{\substack{j=1\\ j\neq i}}^{N} \alpha_{ij} P_i x_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\alpha_{ij}} x_j^T(t-d_{ij})A_{ij}^T P_i A_{ij} x_j(t-d_{ij}) \tag{15}$$

$$\begin{aligned} x_i^T \sum_{k=1}^{m_i} \operatorname{sat} u_{ik} B_{ik}^T P_i x_i + x_i^T P_i \sum_{k=1}^{m_i} \operatorname{sat} u_{ik} B_{ik} x_i &\le x_i^T \sum_{k=1}^{m_i} \gamma_{ik} P_i x_i + x_i^T \sum_{k=1}^{m_i} \frac{1}{\gamma_{ik}} (\operatorname{sat} u_{ik})^2 B_{ik}^T P_i B_{ik} x_i \\ &\le x_i^T \sum_{k=1}^{m_i} \gamma_{ik} P_i x_i + x_i^T \sum_{k=1}^{m_i} \frac{U_{ik}^2}{\gamma_{ik}} B_{ik}^T P_i B_{ik} x_i. \end{aligned} \tag{16}$$

Substituting the above relations into (13) yields

$$\begin{aligned} \dot{V}(x_i(t),t) \le{}& \sum_{i=1}^{N} \Bigg\{ x_i^T(A_i^T P_i + P_i A_i)x_i + x_i^T \sum_{j=1}^{N} \beta_{ij} P_i x_i + \sum_{j=1}^{N} \frac{1}{\beta_{ij}} f_{ij}^T P_i f_{ij} \\ &+ x_i^T \sum_{\substack{j=1\\ j\neq i}}^{N} \alpha_{ij} P_i x_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\alpha_{ij}} x_j^T(t-d_{ij})A_{ij}^T P_i A_{ij} x_j(t-d_{ij}) + x_i^T \sum_{k=1}^{m_i} \gamma_{ik} P_i x_i \\ &+ x_i^T \sum_{k=1}^{m_i} \frac{U_{ik}^2}{\gamma_{ik}} B_{ik}^T P_i B_{ik} x_i + \sum_{\substack{j=1\\ j\neq i}}^{N} x_j^T \left[\frac{1}{\alpha_{ij}} A_{ij}^T P_i A_{ij} + \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 I \right] x_j \\ &- \sum_{\substack{j=1\\ j\neq i}}^{N} x_j^T(t-d_{ij}) \left[\frac{1}{\alpha_{ij}} A_{ij}^T P_i A_{ij} + \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 I \right] x_j(t-d_{ij}) \Bigg\} \\ \le{}& \sum_{i=1}^{N} \Bigg\{ x_i^T(A_i^T P_i + P_i A_i)x_i + x_i^T \left(\sum_{j=1}^{N} \beta_{ij} + \sum_{\substack{j=1\\ j\neq i}}^{N} \alpha_{ij} + \sum_{k=1}^{m_i} \gamma_{ik} \right) P_i x_i + \sum_{j=1}^{N} \frac{q_i}{\beta_{ij}} \|f_{ij}^T\| \, \|f_{ij}\| \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\alpha_{ij}} q_i x_j^T A_{ij}^T A_{ij} x_j + x_i^T \sum_{k=1}^{m_i} \frac{q_i U_{ik}^2}{\gamma_{ik}} B_{ik}^T B_{ik} x_i \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 x_j^T x_j - \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 x_j^T(t-d_{ij}) x_j(t-d_{ij}) \Bigg\} \\ \le{}& \sum_{i=1}^{N} \Bigg\{ x_i^T(A_i^T P_i + P_i A_i)x_i + x_i^T \left(\sum_{j=1}^{N} \beta_{ij} + \sum_{\substack{j=1\\ j\neq i}}^{N} \alpha_{ij} + \sum_{k=1}^{m_i} \gamma_{ik} \right) P_i x_i + \sum_{j=1}^{N} \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 x_j^T x_j \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_i}{\alpha_{ij}} x_j^T A_{ij}^T A_{ij} x_j + x_i^T \sum_{k=1}^{m_i} \frac{q_i U_{ik}^2}{\gamma_{ik}} B_{ik}^T B_{ik} x_i \Bigg\} \\ <{}& \sum_{i=1}^{N} x_i^T \Bigg\{ -Q_i + \sum_{j=1}^{N} \frac{q_j}{\beta_{ji}} \varepsilon_{ji}^2 I + \left(\sum_{j=1}^{N} \beta_{ij} + \sum_{\substack{j=1\\ j\neq i}}^{N} \alpha_{ij} + \sum_{k=1}^{m_i} \gamma_{ik} \right) q_i I \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_j}{\alpha_{ji}} A_{ji}^T A_{ji} + \sum_{k=1}^{m_i} \frac{q_i U_{ik}^2}{\gamma_{ik}} B_{ik}^T B_{ik} \Bigg\} x_i = 0, \end{aligned} \tag{17}$$

where inequalities (2), (11), and the following relations are used.

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \frac{q_i}{\beta_{ij}} \varepsilon_{ij}^2 x_j^T x_j = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{q_j}{\beta_{ji}} \varepsilon_{ji}^2 x_i^T x_i \tag{18}$$

$$\sum_{i=1}^{N} \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_i}{\alpha_{ij}} x_j^T A_{ij}^T A_{ij} x_j = \sum_{i=1}^{N} \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_j}{\alpha_{ji}} x_i^T A_{ji}^T A_{ji} x_i. \tag{19}$$

Therefore, it is seen that condition (5) can guarantee that $\dot{V}(x_i(t), t)$ is negative and that the large-scale system $\boldsymbol{S}$ is robustly stable. Thus, the proof is completed. □

Remark 1. Recently, the robust stability problem for system (1) was studied in [11]. Stability conditions were developed to ensure the robust stability of system (1). We rewrite those conditions as follows.

$$2\mu(A_i) + \varepsilon_{ii}^2 + 2N_i + m_i + 1 + \lambda_1\left(\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\mu(A_i)}{\mu(A_j)}(A_{ji}^TA_{ji} + \varepsilon_{ji}^2 I) + \sum_{k=1}^{m_i}U_{ik}^2B_{ik}^TB_{ik}\right) < 0, \quad i = 1, 2, \dots, N, \tag{20}$$

where N_i denotes the number of $A_{ij} \neq 0$ corresponding to the ith subsystem with $j = 1, 2, \dots, N$. If we choose $q_i = \frac{-1}{\mu(A_i)}$, $q_j = \frac{-1}{\mu(A_j)}$, $\alpha_{ij} = \beta_{ij} = \gamma_{ik} = 1$, then condition (5) becomes

$$A_i^T + A_i + \sum_{j=1}^{N}\frac{\mu(A_i)}{\mu(A_j)}\varepsilon_{ji}^2 I + (2N_i + m_i + 1)I + \sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\mu(A_i)}{\mu(A_j)}A_{ji}^TA_{ji} + \sum_{k=1}^{m_i}U_{ik}^2B_{ik}^TB_{ik} < 0. \tag{21}$$

Since condition (5) infers that

$$\lambda_1\left[A_i^T + A_i + \sum_{j=1}^{N}\frac{\mu(A_i)}{\mu(A_j)}\varepsilon_{ji}^2 I + (2N_i + m_i + 1)I + \sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\mu(A_i)}{\mu(A_j)}A_{ji}^TA_{ji} + \sum_{k=1}^{m_i}U_{ik}^2B_{ik}^TB_{ik}\right] < 0, \tag{22}$$

we have

$$\begin{aligned}
&\lambda_1\left[A_i^T + A_i + \sum_{j=1}^{N}\frac{\mu(A_i)}{\mu(A_j)}\varepsilon_{ji}^2 I + (2N_i + m_i + 1)I + \sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\mu(A_i)}{\mu(A_j)}A_{ji}^TA_{ji} + \sum_{k=1}^{m_i}U_{ik}^2B_{ik}^TB_{ik}\right]\\
&\quad = \lambda_1\left[A_i^T + A_i + \sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\mu(A_i)}{\mu(A_j)}(A_{ji}^TA_{ji} + \varepsilon_{ji}^2 I) + \sum_{k=1}^{m_i}U_{ik}^2B_{ik}^TB_{ik}\right] + 2N_i + m_i + 1 + \varepsilon_{ii}^2\\
&\quad \leq \lambda_1(A_i^T + A_i) + \lambda_1\left[\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\mu(A_i)}{\mu(A_j)}(A_{ji}^TA_{ji} + \varepsilon_{ji}^2 I) + \sum_{k=1}^{m_i}U_{ik}^2B_{ik}^TB_{ik}\right] + 2N_i + m_i + 1 + \varepsilon_{ii}^2\\
&\quad = 2\mu(A_i) + \varepsilon_{ii}^2 + 2N_i + m_i + 1 + \lambda_1\left(\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\mu(A_i)}{\mu(A_j)}(A_{ji}^TA_{ji} + \varepsilon_{ji}^2 I) + \sum_{k=1}^{m_i}U_{ik}^2B_{ik}^TB_{ik}\right).
\end{aligned} \tag{23}$$

This means that condition (21) is sharper than condition (20). Furthermore, condition (21) is only a special case of condition (5). Therefore, one can conclude that the obtained condition (5) improves condition (20).

Remark 2. In fact, there are many free variables in condition (5). How to select them such that condition (5) has the best result is an open problem. Simple choices for these free variables are

$$q_i = q_j, \qquad \alpha_{ij} = \|A_{ij}\|, \qquad \beta_{ij} = \varepsilon_{ij}, \quad \text{and} \quad \gamma_{ik} = U_{ik}\|B_{ik}\|.$$

Then we have the following corollary.

Corollary 1. *The large-scale uncertain bilinear time-delay system* (1) *with constraints* (3) *is robustly stable if*

$$A_i^T + A_i + \left[\sum_{j=1}^{N}(\varepsilon_{ji} + \varepsilon_{ij}) + \sum_{\substack{j=1\\ j\neq i}}^{N}\|A_{ij}\| + \sum_{k=1}^{m_i}U_{ik}\|B_{ik}\|\right] I + \sum_{\substack{j=1\\ j\neq i}}^{N}\frac{1}{\|A_{ji}\|}A_{ji}^TA_{ji} + \sum_{k=1}^{m_i}\frac{U_{ik}}{\|B_{ik}\|}B_{ik}^TB_{ik} < 0, \tag{24}$$

where $i = 1, 2, \dots, N$. We find that this explicit condition can obtain better results than [11].

Remark 3. The result obtained can be applied to the stability analysis for large-scale uncertain time-delay systems. Let $B_{ik} = 0$ for all i and k. Then (1) becomes the following large-scale uncertain time-delay system:

$$S_i : \dot{x}_i(t) = A_i x_i(t) + \sum_{j=1}^{N} f_{ij}(x_j(t - d_{ij}), t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij} x_j(t - d_{ij}), \quad i = 1, 2, \ldots, N. \tag{25}$$

Then, according to Corollary 1, we can obtain the following result without proof.

Corollary 2. *The large-scale uncertain time-delay system described by (25) is robustly stable if, for $i = 1, 2, \ldots, N$, the following condition is met.*

$$A_i^T + A_i + \left[\sum_{j=1}^{N} (\varepsilon_{ji} + \varepsilon_{ij}) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \|A_{ij}\| \right] I + \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{1}{\|A_{ji}\|} A_{ji}^T A_{ji} < 0. \tag{26}$$

3. Large-scale bilinear internal systems

Consider a homogeneous large-scale bilinear interval system $\tilde{\boldsymbol{S}}$, which is described as an interconnection of N subsystems $\tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_N$ represented by

$$\tilde{S}_i : \dot{x}_i(t) = \tilde{A}_i x_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \tilde{A}_{ij} x_j(t - d_{ij}) + \sum_{k=1}^{m_i} \mathrm{sat}\, u_{ik}(t) \tilde{B}_{ik} x_i(t), \quad i = 1, 2, \ldots, N, \tag{27}$$

where $x_i(\cdot) \in \boldsymbol{R}^n$, $d_{ij} \geq 0$, and $u_{ik}(\cdot) \in \boldsymbol{R}^m$ are the same as those in system (1), and $\tilde{A}_i = [\tilde{a}_{ipq}]$, $\tilde{A}_{ij} = [\tilde{a}_{ijpq}]$, and $\tilde{B}_{ik} = [\tilde{b}_{ikpq}]$ are interval matrices with appropriate dimensions, having the following properties:

$$\tilde{A}_i \in N[U_i, V_i] \quad \text{with } U_i = [u_{ipq}],\ V_i = [v_{ipq}]. \tag{28}$$

$$\tilde{A}_{ij} \in N[U_{ij},\ V_{ij}] \quad \text{with } U_{ij} = [u_{ijpq}],\ V_{ij} = [v_{ijpq}] \tag{29}$$

$$\tilde{B}_{ik} \in N[E_{ik},\ F_{ik}] \quad \text{with } E_{ik} = [e_{ikpq}],\ F_{ik} = [f_{ikpq}]. \tag{30}$$

Functions $N[U_i,\ V_i]$, $N[U_{ij},\ V_{ij}]$, and $N[E_{ik},\ F_{ik}]$ present the set of all matrices $\tilde{A}_i$, $\tilde{A}_{ij}$, and $\tilde{B}_{ik}$ satisfying

$$u_{ipq} \leq \tilde{a}_{ipq} \leq v_{ipq}, \qquad u_{ijpq} \leq \tilde{a}_{ij} \leq v_{ijpq}, \qquad e_{ikpq} \leq \tilde{b}_{ikpq} \leq f_{ikpq}, \quad p, q = 1, 2, \ldots, n. \tag{31}$$

Define

$$\hat{A}_i = [\hat{a}_{ipq}] \equiv \frac{U_i + V_i}{2} \quad \text{and} \quad L_i = [l_{ipq}] \equiv \frac{V_i - U_i}{2}, \quad p, q = 1, 2, \ldots, n. \tag{32}$$

$$\hat{A}_{ij} = [\hat{a}_{ijpq}] \equiv \frac{U_{ij} + V_{ij}}{2} \quad \text{and} \quad M_{ij} = [m_{ijpq}] \equiv \frac{V_{ij} - U_{ij}}{2}, \quad p,\ q = 1, 2, \ldots, n. \tag{33}$$

$$\hat{B}_{ik} = [\hat{b}_{ikpq}] \equiv \frac{E_{ik} + F_{ik}}{2} \quad \text{and} \quad N_{ik} = [n_{ikpq}] \equiv \frac{F_{ik} - E_{ik}}{2}, \quad p,\ q = 1, 2, \ldots, n. \tag{34}$$

Then system (27) can be represented as

$$\tilde{S}_i : \dot{x}_i(t) = (\hat{A}_i + \Delta\hat{A}_i) x_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} (\hat{A}_{ij} + \Delta\hat{A}_{ij}) x_j(t - d_{ij}) + \sum_{k=1}^{m_i} \mathrm{sat}\, u_{ik}(t) (\hat{B}_{ik} + \Delta\hat{B}_{ik}) x_i(t), \tag{35}$$

where $\Delta\hat{A}_i$, $\Delta\hat{A}_{ij}$, and $\Delta\hat{B}_{ik}$ denote parametric uncertainties with the following properties:

$$|\Delta\hat{A}_i| \leq L_i, \qquad |\Delta\hat{A}_{ij}| < M_{ij}, \quad \text{and} \quad |\Delta\hat{B}_{ik}| < N_{ik}, \tag{36}$$

which means that $|\Delta\hat{A}_i| = [|\Delta\hat{a}_{ipq}|]$, $|\Delta\hat{A}_{ij}| = [|\Delta\hat{a}_{ijpq}|]$, $|\Delta\hat{B}_{ik}| = [|\Delta\hat{b}_{ikpq}|]$ and $|\Delta\hat{a}_{ipq}| \leq l_{ipq}$, $|\Delta\hat{a}_{ijpq}| \leq m_{ijpq}$, $|\Delta\hat{b}_{ikpq}| \leq n_{ikpq}$, for $p,\ q = 1, 2, \ldots, n$. Then, due to the well-known facts that $\|A\| \leq \||A|\|$ and $\mu(A) \leq \mu(|A|)$, we have

$$\|\Delta\hat{A}_{ij}\| \leq \||\Delta\hat{A}_{ij}|\| \leq \|M_{ij}\|, \qquad \|\Delta\hat{B}_{ik}\| \leq \||\Delta\hat{B}_{ik}|\| \leq \|N_{ik}\|, \quad \text{and} \quad \mu(\Delta\hat{A}_i) \leq \mu(|\Delta\hat{A}_i|) \leq \mu(L_i). \tag{37}$$

Then in light of Theorem 1 and (36)–(37), we have the following result.

Theorem 2. *If there exist positive constants α_{ij}, q_i, and β_{ik} such that the following condition is satisfied for $i = 1, 2, \ldots, N$,*

$$\hat{A}_i^T + \hat{A}_i + 2\mu(L_i)I + \sum_{\substack{j=1\\ j\neq i}}^{N} \alpha_{ij} I + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_j}{\alpha_{ji} q_i} (\hat{A}_{ji}^T \hat{A}_{ji} + 2\|\hat{A}_{ji}\|\,\|M_{ji}\| I + \|M_{ji}\|^2 I)$$

$$+ \sum_{k=1}^{m_i} \beta_{ik} I + \sum_{k=1}^{m_i} \frac{U_{ik}^2}{\beta_{ik}} (\hat{B}_{ik}^T \hat{B}_{ik} + 2\|\hat{B}_{ik}\|\,\|N_{ik}\| I + \|N_{ik}\|^2 I) < 0, \tag{38}$$

then the large-scale bilinear interval system (27) with constraints (3) is robustly stable.

Proof. According to (37), we have

$$\Delta\hat{A}_i^T + \Delta\hat{A}_i \le 2\mu(\Delta\hat{A}_i)I \le 2\mu(|\Delta\hat{A}_i|)I \le 2\mu(L_i)I. \tag{39}$$

Then, from condition (38), it is obvious that matrix $\hat{A}_i + \Delta\hat{A}_i < 0$, which means that $\hat{A}_i + \Delta\hat{A}_i$ is stable. We choose the Lyapunov function $V(x_i(t), t)$ for the large-scale system (27) as

$$V(x_i(t), t) = \sum_{i=1}^{N} V_i(x_i(t), t) = \sum_{i=1}^{N} \left\{ x_i^T P_i x_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \int_{t-d_{ij}}^{t} x_j^T(s) \frac{\tilde{A}_{ij}^T P_i \tilde{A}_{ij}}{\alpha_{ij}} x_j(s) ds \right\}, \tag{40}$$

where P_i satisfies the following Lyapunov equation:

$$(\hat{A}_i + \Delta\hat{A}_i)^T P_i + P_i(\hat{A}_i + \Delta\hat{A}_i) = -Q_i, \quad i = 1, 2, \ldots, N. \tag{41}$$

Here, matrix Q_i is selected as

$$Q_i \equiv q_i \Bigg[\sum_{\substack{j=1\\ j\neq i}}^{N} \alpha_{ij} I + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_j}{\alpha_{ji} q_i} (\hat{A}_{ji}^T \hat{A}_{ji} + 2\|\hat{A}_{ji}\|\,\|M_{ji}\| I + \|M_{ji}\|^2 I)$$

$$+ \sum_{k=1}^{m_i} \beta_{ik} I + \sum_{k=1}^{m_i} \frac{U_{ik}^2}{\beta_{ik}} (\hat{B}_{ik}^T \hat{B}_{ik} + 2\|\hat{B}_{ik}\|\,\|N_{ik}\| I + \|N_{ik}\|^2 I) \Bigg], \tag{42}$$

where q_i, α_{ij}, and β_{ij} are arbitrary positive constants. Then, using similar methods as in the proof of Theorem 1, if condition (38) holds, then the solution P_i has the bound

$$P_i < qI, \quad i = 1, 2, \ldots, N. \tag{43}$$

Taking the derivative of $V(x_i(t), t)$ along trajectories of (35) results in

$$\begin{aligned}
\dot{V}(x_i(t), t) &= \sum_{i=1}^{N} \left\{ \dot{x}_i^T P_i x_i + x_i^T P_i \dot{x}_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\alpha_{ij}} x_j^T \hat{A}_{ij}^T P_i \hat{A}_{ij} x_j - \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\alpha_{ij}} x_j^T(t-d_{ij}) \hat{A}_{ij}^T P_i \hat{A}_{ij} x_j(t-d_{ij}) \right\} \\
&= \sum_{i=1}^{N} \Bigg\{ x_i^T [(\hat{A}_i + \Delta\hat{A}_i)^T P_i + P_i(\hat{A}_i + \Delta\hat{A}_i)] x_i + \sum_{\substack{j=1\\ j\neq i}}^{N} x_j^T(t-d_{ij}) \tilde{A}_{ij}^T P_i x_i \\
&\quad + x_i^T P_i \sum_{\substack{j=1\\ j\neq i}}^{N} \tilde{A}_{ij} x_j(t-d_{ij}) + x_i^T \left[\sum_{k=1}^{m_i} \operatorname{sat} u_{ik} \tilde{B}_{ik}^T P_i + P_i \sum_{k=1}^{m_i} \operatorname{sat} u_{ik} \tilde{B}_{ik} \right] x_i \\
&\quad + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\alpha_{ij}} x_j^T \hat{A}_{ij}^T P_i \hat{A}_{ij} x_j - \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\alpha_{ij}} x_j^T(t-d_{ij}) \hat{A}_{ij}^T P_i \hat{A}_{ij} x_j(t-d_{ij}) \Bigg\}.
\end{aligned} \tag{44}$$

By similar methods as in the proof of Theorem 1, we obtain

$$\begin{aligned}
\dot{V}(x_i(t), t) &\le \sum_{i=1}^{N} \Bigg\{ x_i^T [(\hat{A}_i + \Delta\hat{A}_i)^T P_i + P_i(\hat{A}_i + \Delta\hat{A}_i)] x_i + x_i^T \sum_{\substack{j=1\\ j\neq i}}^{N} \alpha_{ij} P_i x_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\alpha_{ij}} x_j^T \tilde{A}_{ij}^T P_i \tilde{A}_{ij} x_j \\
&\quad + x_i^T \sum_{k=1}^{m_i} \beta_{ik} P_i x_i + x_i^T \sum_{k=1}^{m_i} \frac{U_{ik}^2}{\beta_{ik}} \tilde{B}_{ik}^T P_i \tilde{B}_{ik} x_i
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{\alpha_{ij}}x_j^T\hat{A}_{ij}^TP_i\hat{A}_{ij}x_j-\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{\alpha_{ij}}x_j^T(t-d_{ij})\hat{A}_{ij}^TP_i\hat{A}_{ij}x_j(t-d_{ij})\Bigg\}\\
&\le\sum_{i=1}^{N}x_i^T\Bigg\{[(\hat{A}_i+\Delta\hat{A}_i)^TP_i+P_i(\hat{A}_i+\Delta\hat{A}_i)]+\sum_{\substack{j=1\\j\neq i}}^{N}\alpha_{ij}P_i+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{\alpha_{ji}}\tilde{A}_{ji}^TP_j\tilde{A}_{ji}\\
&+\sum_{k=1}^{m_i}\beta_{ik}P_i+\sum_{k=1}^{m_i}\frac{U_{ik}^2}{\beta_{ik}}\tilde{B}_{ik}^TP_i\tilde{B}_{ik}\Bigg\}x_i\\
&<\sum_{i=1}^{N}x_i^T\Bigg\{-Q_i+q_i\Bigg[\sum_{\substack{j=1\\j\neq i}}^{N}\alpha_{ij}+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{q_j}{\alpha_{ji}q_i}\tilde{A}_{ji}^T\tilde{A}_{ji}+\sum_{k=1}^{m_i}\beta_{ik}+\sum_{k=1}^{m_i}\frac{U_{ik}^2}{\beta_{ik}}\tilde{B}_{ik}^T\tilde{B}_{ik}\Bigg]\Bigg\}x_i.
\end{aligned}
\tag{45}
$$

Furthermore, we also have

$$
\begin{aligned}
\tilde{A}_{ji}^T\tilde{A}_{ji}&=(\hat{A}_{ji}^T+\Delta\hat{A}_{ji}^T)(\hat{A}_{ji}+\Delta\hat{A}_{ji})=\hat{A}_{ji}^T\hat{A}_{ji}+\hat{A}_{ji}^T\Delta\hat{A}_{ji}+\Delta\hat{A}_{ji}^T\hat{A}_{ji}+\Delta\hat{A}_{ji}^T\Delta\hat{A}_{ji}\\
&\le\hat{A}_{ji}^T\hat{A}_{ji}+(2\|\hat{A}_{ji}^T\|\,\|\Delta\hat{A}_{ji}\|+\|\Delta\hat{A}_{ji}\|^2)I\le\hat{A}_{ji}^T\hat{A}_{ji}+(2\|\hat{A}_{ji}^T\|\,\|M_{ji}\|+\|M_{ji}\|^2)I
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
\tilde{B}_{ik}^T\tilde{B}_{ik}&=(\hat{B}_{ik}^T+\Delta\hat{B}_{ik}^T)(\hat{B}_{ik}+\Delta\hat{B}_{ik})\le\hat{B}_{ik}^T\hat{B}_{ik}+(2\|\hat{B}_{ik}\|\,\|\Delta\hat{B}_{ik}\|+\|\Delta\hat{B}_{ik}\|^2)I\\
&\le\hat{B}_{ik}^T\hat{B}_{ik}+(2\|\hat{B}_{ik}\|\,\|N_{ik}\|+\|N_{ik}\|^2)I.
\end{aligned}
\tag{47}
$$

Substituting (46) and (47) into (45) yields

$$
\begin{aligned}
\dot{V}(x_i(t),t)&<\sum_{i=1}^{N}x_i^T\Bigg\{-Q_i+q_i\Bigg[+\sum_{\substack{j=1\\j\neq i}}^{N}\alpha_{ij}I+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{q_j}{\alpha_{ji}q_i}(\hat{A}_{ji}^T\hat{A}_{ji}+2\|\hat{A}_{ji}\|\,\|M_{ji}\|I+\|M_{ji}\|^2I)\Bigg]\\
&\quad+q_i\Bigg[\sum_{k=1}^{m_i}\beta_{ik}I+\sum_{k=1}^{m_i}\frac{U_{ik}^2}{\beta_{ik}}(\hat{B}_{ik}^T\hat{B}_{ik}+2\|\hat{B}_{ik}\|\,\|N_{ik}\|I+\|N_{ik}\|^2I)\Bigg]\Bigg\}x_i\\
&=\sum_{i=1}^{N}x_i^T\{-Q_i+Q_i\}x_i<0,
\end{aligned}
\tag{48}
$$

where relation (37) is used. From (48), it is seen that if condition (38) holds then $\dot{V}(x_i(t),t)$ is negative, which can ensure the robust stability of the large-scale bilinear interval system $\tilde{S}$. Thus, the proof is completed. □

Remark 4. Note that the Lyapunov equation (41) is unsolvable due to the parametric uncertainties $\Delta\hat{A}_i$. However, by using the upper bound of P_i, it is not necessary to solve any Lyapunov equation for the robust condition (38).

Remark 5. Define

$$\bar{A}_{ij}=[\bar{a}_{ijpq}]\quad\text{with } \bar{a}_{ijpq}\equiv\max(|u_{ijpq}|,\ |v_{ijpq}|)\tag{49}$$

$$\bar{B}_{ik}=[\bar{b}_{ikpq}]\quad\text{with } \bar{b}_{kij}\equiv\max(|e_{ikpq}|,\ |f_{ikpq}|).\tag{50}$$

Let $\tilde{N}_i$ denote the number of $\tilde{A}_{ij}\neq 0$ corresponding to the ith subsystem with $j=1,2,\ldots,N$. Then the following condition was proposed in [11] to guarantee the robust stability of system (35).

$$2[\mu(\hat{A}_i)+\mu(L_i)]+\tilde{N}_i+m_i+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{\mu(\hat{A}_i)+\mu(L_i)}{\mu(\hat{A}_j)+\mu(L_j)}\|\bar{A}_{ji}\|^2+\sum_{k=1}^{m_i}U_{ik}^2\|\bar{B}_{ik}\|^2<0,\quad i=1,2,\ldots,N.\tag{51}$$

If the constants q_i, q_j, α_{ij}, α_{ji}, and β_{ij} are selected, respectively, as

$$q_i=\frac{1}{\mu(\hat{A}_j)+\mu(L_i)},\qquad \alpha_{ij}=\alpha_{ji}=\beta_{ij}=1,\tag{52}$$

then condition (38) becomes

$$
\begin{aligned}
&\hat{A}_i^T+\hat{A}_i+2\mu(L_i)I+(\tilde{N}_i+m_i)I+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{\mu(\hat{A}_i)+\mu(L_i)}{\mu(\hat{A}_j)+\mu(L_j)}(\hat{A}_{ji}^T\hat{A}_{ji}+2\|\hat{A}_{ji}\|\,\|M_{ji}\|I+\|M_{ji}\|^2I)\\
&+\sum_{k=1}^{m_i}U_{ik}^2(\hat{B}_{ik}^T\hat{B}_{ik}+2\|\hat{B}_{ik}\|\,\|N_{ik}\|I+\|N_{ik}\|^2I)<0.
\end{aligned}
\tag{53}
$$

Since

$$\hat{A}_i^T + \hat{A}_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{\mu(\hat{A}_i) + \mu(L_i)}{\mu(\hat{A}_j) + \mu(L_j)} \hat{A}_{ji}^T \hat{A}_{ji} + \sum_{k=1}^{m_i} U_{ik}^2 \hat{B}_{ik}^T \hat{B}_{ik}$$
$$\leq \lambda_i \left(\hat{A}_i^T + \hat{A}_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{\mu(\hat{A}_i) + \mu(L_i)}{\mu(\hat{A}_j) + \mu(L_j)} \hat{A}_{ji}^T \hat{A}_{ji} + \sum_{k=1}^{m_i} U_{ik}^2 \hat{B}_{ik}^T \hat{B}_{ik} \right) I$$
$$\leq \lambda_1 \left(\hat{A}_i^T + \hat{A}_i \right) I + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{\mu(\hat{A}_i) + \mu(L_i)}{\mu(\hat{A}_j) + \mu(L_j)} \|\hat{A}_{ji}\|^2 I + \sum_{k=1}^{m_i} U_{ik}^2 \|\hat{B}_{ik}\|^2 I, \tag{54}$$

condition (53) then implies that

$$2[\mu(\hat{A}_i) + \mu(L_i)] + \tilde{N}_i + m_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{\mu(\hat{A}_i) + \mu(L_i)}{\mu(\hat{A}_j) + \mu(L_j)} (\|\hat{A}_{ji}\| + \|M_{ji}\|)^2 + \sum_{k=1}^{m_i} U_{ik}^2 (\|\hat{B}_{ik}\| + \|N_{ij}\|)^2 < 0. \tag{55}$$

From the following properties

$$\|\tilde{A}_{ji}\| \leq \||\tilde{A}_{ji}|\| \leq \|\bar{A}_{ji}\|, \quad \|\tilde{A}_{ji}\| = \|\hat{A}_{ji} + \Delta\hat{A}_{ji}\| \leq \|\hat{A}_{ji}\| + \|\Delta\hat{A}_{ji}\| \leq \|\hat{A}_{ji}\| + \|M_{ji}\| \tag{56}$$
$$\|\tilde{B}_{ij}\| \leq \||\tilde{B}_{ij}|\| \leq \|\bar{B}_{ij}\|, \quad \|\tilde{B}_{ij}\| = \|\hat{B}_{ij} + \Delta\hat{B}_{ij}\| \leq \|\hat{B}_{ij}\| + \|N_{ij}\|, \tag{57}$$

it is seen that, if $\|\hat{A}_{ji}\| + \|M_{ji}\| \leq \|\bar{A}_{ji}\|$ and $\|\hat{B}_{ij}\| + \|N_{ij}\| \leq \|\bar{B}_{ij}\|$, then our result (53) is sharper than (51). Otherwise, the tightness between the obtained result (53) and (51) cannot be compared. Perhaps they can supplement each other for such a case. We also give the following simple choices for these free variables in condition (38):

$$q_i = q_j, \qquad \alpha_{ij} = \|A_{ij}\| + \|M_{ij}\|, \quad \text{and} \quad \beta_{ij} = U_{ik}(\|\hat{B}_{ij}\| + \|N_{ij}\|). \tag{58}$$

Theorem 1, then, becomes the following corollary.

Corollary 3. *The large-scale bilinear interval system (27) with constraints (3) is robustly stable if, for $i = 1, 2, \ldots, N$, the following condition is satisfied.*

$$\hat{A}_i^T + \hat{A}_i + 2\mu(L_i)I + \sum_{\substack{j=1\\ j\neq i}}^{N} (\|\hat{A}_{ij}\| + \|M_{ij}\|)I + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\|\hat{A}_{ji}\| + \|M_{ji}\|} (\hat{A}_{ji}^T \hat{A}_{ji} + 2\|\hat{A}_{ji}\|\,\|M_{ji}\|I + \|M_{ji}\|^2 I)$$
$$+ \sum_{k=1}^{m_i} U_{ik}(\|\hat{B}_{ik}\| + \|N_{ik}\|)I + \sum_{k=1}^{m_i} \frac{U_{ik}}{\|\hat{B}_{ik}\| + \|N_{ik}\|} (\hat{B}_{ik}^T \hat{B}_{ik} + 2\|\hat{B}_{ik}\|\,\|N_{ik}\|I + \|N_{ik}\|^2 I) < 0. \tag{59}$$

Remark 6. Let $\tilde{B}_{ik} = 0$ for all i and k. Then (27) becomes the following large-scale interval time-delay system:

$$\tilde{S}_i : \dot{x}_i(t) = \tilde{A}_i x_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} \tilde{A}_{ij} x_j(t - d_{ij}), \quad i = 1, 2, \ldots, N. \tag{60}$$

Then, according to Corollary 3, we can obtain the following result without proof.

Corollary 4. *The large-scale interval time-delay system (60) is robustly stable if, for $i = 1, 2, \ldots, N$, the following condition is met.*

$$\hat{A}_i^T + \hat{A}_i + 2\mu(L_i)I + \sum_{\substack{j=1\\ j\neq i}}^{N} (\|\hat{A}_{ij}\| + \|M_{ij}\|)I + \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{\|\hat{A}_{ji}\| + \|M_{ji}\|} (\hat{A}_{ji}^T \hat{A}_{ji} + 2\|\hat{A}_{ji}\|\,\|M_{ji}\|I + \|M_{ji}\|^2 I) < 0. \tag{61}$$

4. Illustrative examples

Examples are given below to show the merits of the results obtained.

Example 1. Consider the following large-scale bilinear uncertain time-delay system proposed in [11].

$$\begin{aligned}
\dot{x}_1(t) &= \begin{bmatrix} -5 & 0.5 \\ 0 & -6 \end{bmatrix} x_1(t) + \sum_{j=1}^{3} f_{1j}(x_j(t-d_{1j}), t) + \begin{bmatrix} -0.8 & 0.3 \\ 0 & 0.6 \end{bmatrix} x_2(t-d_{12}) \\
&\quad + \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.6 \end{bmatrix} x_3(t-d_{13}) + \operatorname{sat} u_{11}(t) \begin{bmatrix} 0.3 & -0.1 \\ 0.1 & 0.2 \end{bmatrix} x_1(t) \\
\dot{x}_2(t) &= \begin{bmatrix} -6 & 0.5 \\ 0.3 & -6 \end{bmatrix} x_2(t) + \sum_{j=1}^{3} f_{2j}(x_j(t-d_{2j}), t) + \begin{bmatrix} 0.5 & 0 \\ 0.2 & -0.6 \end{bmatrix} x_1(t-d_{21}) \\
&\quad + \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & -0.4 \end{bmatrix} x_3(t-d_{23}) + \operatorname{sat} u_{21}(t) \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix} x_2(t) \\
\dot{x}_3(t) &= \begin{bmatrix} -7 & 1 \\ 0 & -6 \end{bmatrix} x_3(t) + f_{33}(x_3(t), t) + \begin{bmatrix} -0.8 & 0.3 \\ 0 & -0.6 \end{bmatrix} x_1(t-d_{31}) \\
&\quad + f_{31}(x_1(t-d_{31}), t) + \operatorname{sat} u_{31}(t) \begin{bmatrix} 0.4 & -0.1 \\ 0 & 0.2 \end{bmatrix} x_3(t).
\end{aligned}$$

Assume that $U_{11} = 1.2$, $U_{21} = 1.0$, and $U_{31} = 1.5$. For this case, it is seen that $m_1 = m_2 = m_3 = 1$, $N_1 = 2$, $N_2 = 2$, and $N_3 = 1$. Now, by the stability condition (20), the uncertainty bounds that can guarantee the robust stability of this large-scale system have been estimated in [11] as

$$\varepsilon_{11}^2 + 0.8823\varepsilon_{21}^2 + 0.8529\varepsilon_{31}^2 < 2.6862 \tag{62}$$

$$1.1334\varepsilon_{12}^2 + \varepsilon_{22}^2 < 4.2147 \tag{63}$$

$$1.1724\varepsilon_{13}^2 + 1.0344\varepsilon_{23}^2 + \varepsilon_{33}^2 < 6.7055. \tag{64}$$

From Remark 1, the following uncertainty bounds are obtained by using (21).

$$\varepsilon_{11}^2 + 0.8823\varepsilon_{21}^2 + 0.8529\varepsilon_{31}^2 < 3.0402 \tag{65}$$

$$1.1334\varepsilon_{12}^2 + \varepsilon_{22}^2 < 4.7729 \tag{66}$$

$$1.1724\varepsilon_{13}^2 + 1.0344\varepsilon_{23}^2 + \varepsilon_{33}^2 < 6.7222. \tag{67}$$

Obviously, the presented results are better. Here, assuming that $\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{13} = \varepsilon_{21} = \varepsilon_{22} = \varepsilon_{23} = \varepsilon_{31} = \varepsilon_{33} = 1$, inequality (62) cannot be satisfied. That is, the stability of this system cannot be ensured by [11]. However, inequalities (65)–(67) are met. This means that the proposed condition (21) can guarantee the stability of the system for this case. Furthermore, if we use condition (24), the results are

$$\begin{aligned}
&A_1^T + A_1 + \left[\sum_{j=1}^{3} (\varepsilon_{j1}^{+}\varepsilon_{1j}) + \sum_{\substack{j=1 \\ j\neq i}}^{3} \|A_{1j}\| + U_{11}\|B_{11}\| \right] I + \sum_{\substack{j=1 \\ j\neq i}}^{3} \frac{A_{j1}^T A_{j1}}{\|A_{j1}\|} + \frac{U_{11} B_{11}^T B_{11}}{\|B_{11}\|} \\
&= \begin{bmatrix} -0.5156 & 0.0157 \\ 0.0157 & -2.8112 \end{bmatrix} < 0 \\
&A_2^T + A_2 + \left[\sum_{j=1}^{3} (\varepsilon_{j2}^{+}\varepsilon_{2j}) + \sum_{\substack{j=1 \\ j\neq i}}^{3} \|A_{2j}\| + U_{21}\|B_{21}\| \right] I + \sum_{\substack{j=1 \\ j\neq i}}^{3} \frac{A_{j2}^T A_{j2}}{\|A_{j2}\|} + \frac{U_{21} B_{21}^T B_{21}}{\|B_{21}\|} \\
&= \begin{bmatrix} -3.5378 & 0.5322 \\ 0.5322 & -3.9165 \end{bmatrix} < 0 \\
&A_3^T + A_3 + \left[\sum_{j=1}^{3} (\varepsilon_{j3}^{+}\varepsilon_{3j}) + \sum_{\substack{j=1 \\ j\neq i}}^{3} \|A_{3j}\| + U_{31}\|B_{31}\| \right] I + \sum_{\substack{j=1 \\ j\neq i}}^{3} \frac{A_{j3}^T A_{j3}}{\|A_{j3}\|} + \frac{U_{31} B_{31}^T B_{31}}{\|B_{31}\|} \\
&= \begin{bmatrix} -5.2977 & 1.1446 \\ 1.1446 & -3.2946 \end{bmatrix} < 0.
\end{aligned}$$

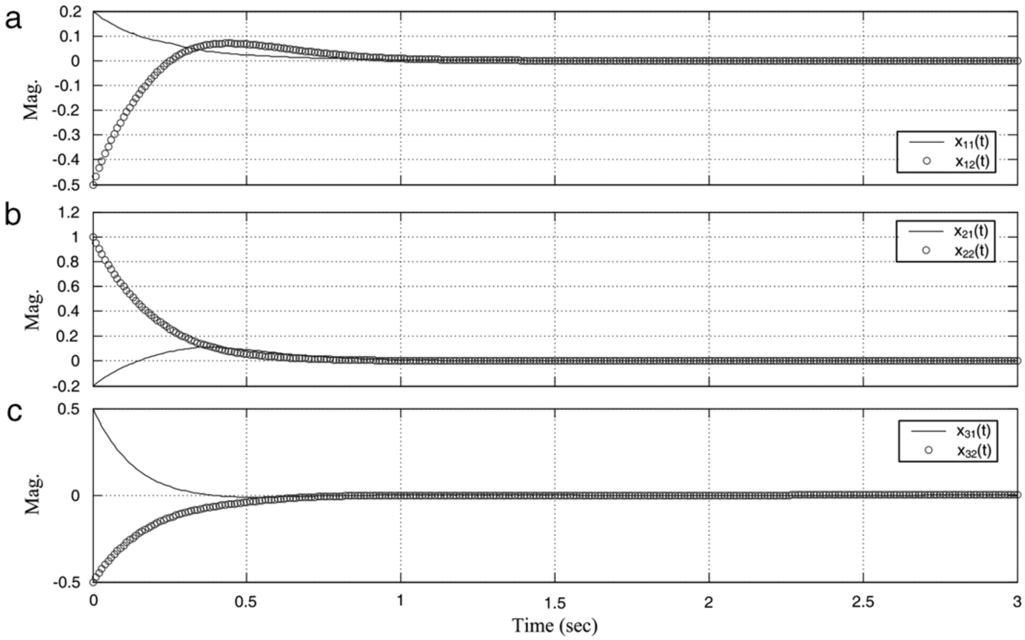

Fig. 1. Simulation results for example 1: (a) the trajectory of state $x_1(t)$, (b) the trajectory of state $x_2(t)$, (c) the trajectory of state $x_3(t)$.

The robust stability of this system can also be ensured. Therefore, the presented condition (5) is indeed better than (20).

Now, let the initial values of the states be $x_1(t) = [0.2 \quad -0.5]^T$, $x_2(t) = [-0.2 \quad 1]^T$, and $x_3(t) = [0.5 \quad -0.5]^T$ for $t < 0$, with corresponding time-delay constants $d_{12} = 0.2$, $d_{13} = 0.3$, $d_{21} = 0.2$, $d_{23} = 0.3$, and $d_{31} = 0.4$. The inputs and nonlinear uncertainties are respectively given as

$$\begin{aligned}
&u_{11}(t) = 1.2\sin 4t, \qquad u_{21} = \sin 2t, \qquad u_{31} = 1.5\cos t, \qquad f_{11}(x_1(t), t) = \cos t\, x_1(t),\\
&f_{12}(x_2(t-d_{12}), t) = \sin t\, x_2(t-d_{12}), \qquad f_{13}(x_3(t-d_{13}), t) = -\cos 2t\, x_3(t-d_{13}),\\
&f_{21}(x_1(t-d_{21}), t) = \cos 2t\, x_1(t-d_{21}), \qquad f_{22}(x_2(t), t) = t + 1/|t| + 1\, x_2(t),\\
&f_{23}(x_3(t-d_{23}), t) = \sin 4t\, x_3(t-d_{23}), \qquad f_{33}(x_3(t), t) = -\sin 3t\, x_3(t),\\
&f_{31}(x_1(t-d_{31}), t) = -\cos t\, x_1(t-d_{31}).
\end{aligned}$$

Then, simulation results for the trajectories of all states are given in Fig. 1. It is observed that all states are regulated to zero asymptotically irrespective of those nonlinear uncertainties.

Example 2. Consider the following large-scale bilinear interval system that is modified slightly from that given in [11]:

$$\begin{aligned}
\dot{x}_1(t) &= \begin{bmatrix} [-6.5 \quad -6] & [-0.5 \quad 0.3] \\ 0 & [-5.5 \quad -5] \end{bmatrix} x_1(t) + \begin{bmatrix} [-0.5 \quad -0.1] & [-0.2 \quad 0] \\ [-0.1 \quad 0.3] & [-0.5 \quad -0.1] \end{bmatrix} x_2(t-d_{12})\\
&\quad + \begin{bmatrix} [0.3 \quad 0.5] & [-0.2 \quad 0] \\ [0 \quad 0.2] & [-0.5 \quad -0.2] \end{bmatrix} x_3(t-d_{13}) + \operatorname{sat} u_{11}(t) \begin{bmatrix} [-0.1 \quad 0.1] & 0 \\ [0.2 \quad 0.4] & [-0.3 \quad -0.1] \end{bmatrix} x_1(t)\\
\dot{x}_2(t) &= \begin{bmatrix} [-6.5 \quad -6] & [-0.5 \quad -0.3] \\ [0.1 \quad 0.2] & [-7.5 \quad -6.5] \end{bmatrix} x_2(t) + \begin{bmatrix} [-0.5 \quad -0.2] & [-0.2 \quad 0] \\ [-0.3 \quad -0.1] & [0.4 \quad 0.6] \end{bmatrix} x_3(t-d_{23})\\
&\quad + \operatorname{sat} u_{21}(t) \begin{bmatrix} [-0.2 \quad 0.1] & [0 \quad 0.1] \\ [-0.2 \quad 0] & [0.3 \quad 0.4] \end{bmatrix} x_2(t) + \operatorname{sat} u_{22}(t) \begin{bmatrix} \begin{bmatrix} -0.1 & 0.1 \\ -0.2 & 0.1 \end{bmatrix} & \begin{matrix} 0 \\ [0.3 \quad 0.6] \end{matrix} \end{bmatrix} x_2(t)\\
\dot{x}_3(t) &= \begin{bmatrix} [-3.25 \quad -2.75] & [0.85 \quad 1.15] \\ [-0.55 \quad -0.45] & [-5.5 \quad -5.2] \end{bmatrix} x_3(t) + \begin{bmatrix} [0.5 \quad 0.6] & [0.2 \quad 0.3] \\ [-0.1 \quad 0] & [-0.5 \quad -0.3] \end{bmatrix} x_1(t-d_{31})\\
&\quad + \begin{bmatrix} [-0.4 \quad -0.2] & [-0.3 \quad -0.2] \\ [0.1 \quad 0.2] & [0.2 \quad 0.4] \end{bmatrix} x_2(t-d_{32})\\
&\quad + \operatorname{sat} u_{31}(t) \begin{bmatrix} [-0.2 \quad 0.1] & [-0.2 \quad 0] \\ [0.2 \quad 0.3] & [0.3 \quad 0.4] \end{bmatrix} x_3(t) + \operatorname{sat} u_{32}(t) \begin{bmatrix} [-0.1 \quad 0.1] & [-0.2 \quad -0.1] \\ 0 & [-0.5 \quad -0.3] \end{bmatrix} x_3(t).
\end{aligned}$$

It is seen that $m_1 = 1, m_2 = 2, m_3 = 2, N_1 = 2, N_2 = 1$, and $N_3 = 2$. For this case, let $U_{11} = 2.5, U_{21} = 1.5, U_{22} = 1.5, U_{31} = 0.6$, and $U_{32} = 0.6$. Then, according to the stability condition (51), we have

$$2[\mu\left(\hat{A}_1\right) + \mu(L_1)] + \tilde{N}_1 + m_1 + \left(\frac{\mu\left(\hat{A}_1\right) + \mu(L_1)}{\mu\left(\hat{A}_3\right) + \mu(L_3)}\|\bar{A}_{31}\|^2 + \frac{\mu\left(\hat{A}_1\right) + \mu(L_1)}{\mu\left(\hat{A}_2\right) + \mu(L_2)}\|\bar{A}_{21}\|^2 + \bar{\beta}_1\right) = -3.0088$$

$$2[\mu\left(\hat{A}_2\right) + \mu(L_2)] + \tilde{N}_2 + m_2 + \left(\frac{\mu\left(\hat{A}_2\right) + \mu(L_2)}{\mu\left(\hat{A}_1\right) + \mu(L_1)}\|\bar{A}_{12}\|^2 + \frac{\mu\left(\hat{A}_2\right) + \mu(L_2)}{\mu\left(\hat{A}_3\right) + \mu(L_3)}\|\bar{A}_{32}\|^2 + \bar{\beta}_2\right) = -4.3286$$

$$2[\mu\left(\hat{A}_3\right) + \mu(L_3)] + \tilde{N}_3 + m_3 + \left(\frac{\mu\left(\hat{A}_3\right) + \mu(L_3)}{\mu\left(\hat{A}_1\right) + \mu(L_1)}\|\bar{A}_{13}\|^2 + \frac{\mu\left(\hat{A}_3\right) + \mu(L_3)}{\mu\left(\hat{A}_2\right) + \mu(L_2)}\|\bar{A}_{23}\|^2 + \bar{\beta}_3\right) = 0.1647,$$

where $\bar{\beta}_i = \sum_{i=1}^{m_i} U_{ik}^2\|B_{ik}\|^2, i = 1, 2, 3$.

Therefore, the stability of this large-scale bilinear interval system cannot be guaranteed by the condition presented in [11]. Now, we check the stability via condition (59), and obtain

$$\begin{aligned}
&\hat{A}_1^T + \hat{A}_1 + 2\mu(L_1)I + \sum_{\substack{j=1\\ j\neq i}}^{3}(\|\hat{A}_{1j}\| + \|M_{1j}\|)I + \sum_{\substack{j=1\\ j\neq i}}^{3}\frac{1}{\|\hat{A}_{j1}\| + \|M_{j1}\|}(\hat{A}_{j1}^T\hat{A}_{j1} + 2\|\hat{A}_{j1}\|\,\|M_{j1}\|I + \|M_{j1}\|^2 I)\\
&\quad + U_{11}(\|\hat{B}_{11}\| + \|N_{11}\|)I + \frac{U_{11}}{\|\hat{B}_{11}\| + \|N_{11}\|}(\hat{B}_{11}^T\hat{B}_{11} + 2\|\hat{B}_{11}\|\,\|N_{11}\|I + \|N_{11}\|^2 I)\\
&\quad = \begin{bmatrix} -4.6805 & -0.1471 \\ -0.1471 & -2.9534 \end{bmatrix} < 0\\
&\hat{A}_2^T + \hat{A}_2 + 2\mu(L_2)I + \sum_{\substack{j=1\\ j\neq i}}^{3}(\|\hat{A}_{2j}\| + \|M_{2j}\|)I + \sum_{\substack{j=1\\ j\neq i}}^{3}\frac{1}{\|\hat{A}_{j2}\| + \|M_{j2}\|}(\hat{A}_{j2}^T\hat{A}_{j2} + 2\|\hat{A}_{j2}\|\,\|M_{j2}\|I + \|M_{j2}\|^2 I)\\
&\quad + \sum_{k=1}^{2}U_{2k}(\|\hat{B}_{2k}\| + \|N_{2k}\|)I + \sum_{k=1}^{2}\frac{U_{2k}}{\|\hat{B}_{2k}\| + \|N_{2k}\|}(\hat{B}_{2k}^T\hat{B}_{2k} + 2\|\hat{B}_{2k}\|\,\|N_{2k}\|I + \|N_{2k}\|^2 I)\\
&\quad = \begin{bmatrix} -3.3715 & -0.2126 \\ -0.2126 & -4.2660 \end{bmatrix} < 0\\
&\hat{A}_3^T + \hat{A}_3 + 2\mu(L_3)I + \sum_{\substack{j=1\\ j\neq i}}^{3}(\|\hat{A}_{3j}\| + \|M_{3j}\|)I + \sum_{\substack{j=1\\ j\neq i}}^{3}\frac{1}{\|\hat{A}_{j3}\| + \|M_{j3}\|}(\hat{A}_{j3}^T\hat{A}_{j3} + 2\|\hat{A}_{j3}\|\,\|M_{j3}\|I + \|M_{j3}\|^2 I)\\
&\quad + \sum_{k=1}^{2}U_{3k}(\|\hat{B}_{3k}\| + \|N_{3k}\|)I + \sum_{k=1}^{2}\frac{U_{3k}}{\|\hat{B}_{3k}\| + \|N_{3k}\|}(\hat{B}_{3k}^T\hat{B}_{3k} + 2\|\hat{B}_{3k}\|\,\|N_{3k}\|I + \|N_{3k}\|^2 I)\\
&\quad = \begin{bmatrix} -0.3446 & 0.4838 \\ 0.4838 & -4.8182 \end{bmatrix} < 0.
\end{aligned}$$

This system then is indeed robustly stable. In this case, although $\|\hat{A}_{ji}\| + \|N_{ji}\| > \|\bar{A}_{ji}\|$ and $\|\hat{B}_{ij}\| + \|N_{ij}\| > \|\bar{B}_{ij}\|$ for all i, j, condition (59) is satisfied for all i. Therefore, it is better than condition (51) for this case.

5. Conclusions

In this paper, the robust stability test problem for homogeneous bilinear time-delay systems with constrained inputs and uncertainties has been addressed. Both nonlinear and parametric uncertainties have been treated. By using the Lyapunov equation approach associated with linear algebraic techniques and simple upper bounds of the solution of the Lyapunov equation, several delay-independent conditions that guarantee the robust stability of overall systems have been established. It is shown that these results can be applied to solve the stability analysis for large-scale uncertain time-delay systems. Finally, illustrative examples have been given to demonstrate the merits of the present schemes. We believe that this work is helpful for controller design of large-scale bilinear uncertain time-delay systems.

Acknowledgments

The author would like to thank the National Science Council, the Republic of China, for financial support of this research under grants NSC 100-2221-E-230-005 and NSC 100-2221-E-230-009.

References

[1] A. Bibi, Evolutionary transfer functions of bilinear processes with time-varying coefficients, Comput. Math. Appl. 52 (3–4) (2006) 331–338.
[2] G.Y. Tang, C. Li, L. Sun, Optimal tracking control for large-scale interconnected systems with time-delays, Comput. Math. Appl. 53 (1) (2007) 80–86.
[3] W.X. Ma, Engui Fan, Linear superposition principle applying to Hirota bilinear equations, Comput. Math. Appl. 61 (1) (2011) 950–959.
[4] S.W. Churchill, Interaction of chemical reactions and transport. 1. An overview, Ind. Eng. Chem. Res. 44 (14) (2005) 5199–5212.
[5] M. Ekman, Bilinear black-box identification and MPC of the activated sludge process, J. Process Control 18 (7–8) (2008) 643–653.
[6] R.R. Mohler, Nonlinear System Volume II, Applications to Bilinear Control, Prentice Hall, New Jersey, 1991.
[7] H.S. Ali, M. Zasadzinski, H. Rafaralahy, M. Darouach, Robust H^∞ reduced order filtering for uncertain bilinear systems, Automatica 42 (3) (2006) 405–415.
[8] M. Bacic, M. Cannon, B. Kouvaritakis, Constrained control of SISO bilinear system, IEEE Trans. Automat. Control 48 (8) (2003) 1443–1447.
[9] H. Bounit, Comments on the feedback stabilization for bilinear control systems, Appl. Math. Lett. 16 (6) (2003) 847–851.
[10] O. Chabour, J.C. Vivalda, Remark on local and global stabilization of homogeneous bilinear systems, Systems Control Lett. 41 (2) (2000) 141–143.
[11] C.Y. Chen, C.H. Lee, Robust stability of homogeneous large-scale bilinear systems with time delays and uncertainties, J. Process Control 19 (7) (2009) 1082–1090.
[12] M.S. Chen, S.T. Tsao, Exponential stabilization of a class of unstable bilinear systems, IEEE Trans. Automat. Control 45 (5) (2000) 989–992.
[13] Y.P. Chen, J.L. Chang, K.M. Lai, Stability analysis and bang-bang sliding control of a class of single-input bilinear systems, I, IEEE Trans. Automat. Control 45 (11) (2000) 2150–2154.
[14] C.L. Chiang, F.C. Kung, Local stability region in large-scale bilinear systems with decentralized control, J. Control Syst. Technol. 1 (1993) 227–234.
[15] J. Guojun, Stability of bilinear time-delay systems, IMA J. Math. Control Inform. 18 (1) (2001) 53–60.
[16] G. Lu, D.W.C. Ho, Continuous stabilization controllers for singular bilinear systems: The state feedback case, Automatica 42 (2) (2006) 309–314.
[17] S. Hanba, Stabilization of continuous-time homogeneous bilinear systems by constant inputs through nonlinear minimization with applications to the static output feedback stabilization problem, Internat. J. Robust Nonlinear Control 18 (7) (2008) 705–713.
[18] D.W.C. Ho, G. Lu, Y. Zheng, Global stabilization for bilinear systems with time delay, IEE Proc., Control Theory Appl. 149 (1) (2002) 89–94.
[19] G.D. Hu, G.D. Hu, Stabilization of an uncertain large-scale time-dependent bilinear neutral differential system by memory feedback control, IMA J. Math. Control Inform. 18 (1) (2001) 1–18.
[20] H. Jerbi, Global feedback stabilization of new class of bilinear systems, Systems Control Lett. 42 (4) (2001) 313–320.
[21] C.H. Lee, On the stability of uncertain homogeneous bilinear systems subjected to time-delay and constrained inputs, J. Chin. Inst. Eng. 31 (3) (2008) 529–534.
[22] S.I. Niculescu, S. Tarbouriceh, J.M. Dion, L. Dugard, Stability criteria for bilinear systems with delayed state and saturating actuators, in: Proc. of the 34th Conference on Decision & Control, USA, December 13–15, 1995.
[23] H. Shigeru, M. Yoshihiko, Output feedback stabilization of bilinear systems using dead-beat observers, Automatica 37 (6) (2001) 915–920.
[24] D.D. Siljak, M.B. Vukcevic, Decentralized stabilization linear and bilinear large-scale systems, Internat. J. Control 26 (2) (1977) 289–305.
[25] C.W. Tao, W.Y. Wang, M.L. Chan, Design of sliding mode controllers for bilinear systems with time varying uncertainties, IEEE Trans. Syst. Man Cybern. 34 (1) (2004) 639–645.